A Modified Algorithm for Solving Shortest Path Problem with Intuitionistic Fuzzy Arc Lengths

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***Abstract* :** This paper deals with finding the shortest path of a network problem with intuitionistic triangular fuzzy number. A modified algorithm is provided for finding the solution in intuitionistic fuzzy sense. The algorithm is verified by means of an example.

Key words: Intuitionistic Fuzzy Set (IFS), Intuitionistic Fuzzy Shortest Path, Ranking function, Triangular Intuitionistic Fuzzy number (Triangular IFN).

1. *Introduction*

The problem of searching the shortest path is very common and is widely studied on graph theory and optimization areas. Shortest path problems play an important role in routing messages efficiently in networks. Each method has got independent merit of its own address, different types of path searching in different situations. In conventional shortest path problems, it is assumed that decision maker is certain about the parameters (distance, time etc.) between different nodes. But in real life situations, there always exist uncertainty about the parameters between different nodes. In such cases, the parameters are are represented by fuzzy numbers. Fuzzy Shortest Path (FSP) problems have been studied by many researchers and many new algorithms have been developed so far. One such algorithm for finding the shortest path in network flow with fuzzy arc lengths was proposed by Amit Kumar and Manjot Kaur[1]. Here, we have proposed a modified new algorithm for the shortest path with triangular IFN as given in [1].

This paper is organized as follows: Section 2 provides preliminary concepts and definitions. In section 3, the modified algorithm is provided for solving network flow problems with intuitionistic fuzzy arc lengths. The new approach is realized using an illustrative example in section 4 and finally section 5 concludes the paper.\

2. *Preliminary and definitions*

In this section some basic definitions and notations used throughout the paper are presented.

2.1 *Intuitionistic Fuzzy Set*

Let X be an universe of discourse, then an Intuitionistic Fuzzy Set (IFS) A in X is given by

A = { (x , $\mu_A(x)$, $\gamma_A(x)$ / $x \in X$ } where the functions $\mu_A(x) : X \to [0,1]$ and $\gamma_A(x) : X \to [0,1]$ determine the degree of membership and non membership of the element $x \in X$, respectively and for every $x \in X$, $0 \le \mu_A(x) + \gamma_A(x) \le 1$

2.2 *Intuitionistic Fuzzy Number*

Let A = {**x**, $\mu_A(x)$, $\gamma_A(x)$ / $x \in X$ } be an IFS ,then we call ($\mu_A(x)$, $\gamma_A(x)$) an Intuitionistic fuzzy number .We denote it by $(\langle a,b,c\rangle,\langle e,f,g\rangle,\langle p,q,r\rangle)$ where

$$(<a,b,c>,<e,f,g>,<p,q,r>) \in F(I)$$
$$I = [0,1], 0 \le c+g+r \le 1$$

2.3 *Triangular IFN Fuzzy Number and its arithmetic*

A triangular IFN 'A' is denoted by A = { $\mu_A(x)$, $\gamma_A(x)$ / $x \in R$}, where μ_A and γ_A are triangular fuzzy numbers with $\gamma_A \le \mu_A^c$. So a triangular IFN 'A' is given by A = ($\langle x,y,z,\rangle,\langle l,m,n,\rangle,\langle i,j,k\rangle$) with ($\langle i,j,k\rangle \le \langle x,y,z\rangle^c$, $\langle l,m,n\rangle^c$) i.e., either $l \ge y, m \ge z, i \ge m$ and $j \ge n$ or $m \le x, n \le y, j \le l$ and $k \le m$ are membership and non-membership fuzzy numbers of A.

2.4 *Arithmetic operations between two triangular intuitionistic fuzzy numbers*

Let $\tilde{A} = (\langle a_1,b_1,c_1\rangle,\langle e_1,f_1,g_1\rangle,\langle p_1,q_1,r_1\rangle)$

and $\tilde{B} = (\langle a_2,b_2,c_2\rangle,\langle e_2,f_2,g_2\rangle,\langle p_2,q_2,r_2\rangle)$

be two triangular intuitionistic fuzzy numbers, then

$$\tilde{A} \oplus \tilde{B} = \begin{pmatrix} \langle a_1+a_2, b_1+b_2, c_1+c_2\rangle, \langle e_1+e_2, f_1+f_2, g_1+g_2\rangle, \\ \langle p_1+p_2, q_1+q_2, r_1+r_2\rangle \end{pmatrix}$$

$$\tilde{A} \ominus \tilde{B} = \begin{pmatrix} \langle a_1-a_2, b_1-b_2, c_1-c_2\rangle, \langle e_1-e_2, f_1-f_2, g_1-g_2\rangle, \\ \langle p_1-p_2, q_1-q_2, r_1-r_2\rangle \end{pmatrix}$$

2.5 *Ranking function*

A convenient method for comparing of fuzzy number is by use of ranking function. A ranking function

$\Re = F(R) \to R,$ Where F(R) is the set of all fuzzy numbers defined on set of real numbers, maps each fuzzy number into a real number. Let $\widetilde{A}$ and $\widetilde{B}$ be two intuitionistic triangular fuzzy numbers, then

(i) $\widetilde{A} > \widetilde{B}$ if $\Re(\widetilde{A}) > \Re(\widetilde{B})$

(ii) $\widetilde{A} < \widetilde{B}$ if $\Re(\widetilde{A}) < \Re(\widetilde{B})$

(iii) $\widetilde{A} = \widetilde{B}$ if $\Re(\widetilde{A}) = \Re(\widetilde{B})$

For a triangular intuitionistic fuzzy number

$\tilde{A} = \left(\langle a_1, b_1, c_1\rangle, \langle e_1, f_1, g_1\rangle, \langle p_1, q_1, r_1\rangle\right)$

Ranking function is given by

$$\Re(A) = \frac{1}{3}[(a_1 + e_1 + p_1) + (b_1 + f_1 + q_1) + (c_1 + g_1 + r_1)]$$

2.6 Notations

The notations that will be used throughout the paper are as follows:

N{ 1,2,3........,n } : The set all nodes in a network.

Np(j) : The set of all predecessor nodes of node j.

e_{i}: The distance between node i and first node.

e_{ij} : The distance between node i and j.

$\widetilde{e}_i$: The fuzzy distance between node i and first node.

$\widetilde{e}_{ij}$: The fuzzy distance between i and j.

3. Procedure for Intuitionistic FSP length

Step 1 : Assume $\widetilde{e}_1 = \left(\langle 0,0,0\rangle, \langle 0,0,0\rangle, \langle 0,0,0\rangle\right)$

And label the source node (say node1) as

[$\left(\langle 0,0,0\rangle, \langle 0,0,0\rangle, \langle 0,0,0\rangle\right)$, -]

Step 2 : Find $\widetilde{e}_j = \min\{ \widetilde{e}_i \oplus \widetilde{e}_{ij} \,/\, i \in Np(j) \}$

$j \neq 1$, j = 2,3......., n.

Step 3 : If minimum occurs corresponding to unique value of i i.e., i = r then label node j as $[\widetilde{e}_j, r]$.

If minimum occurs corresponding to more than one values of i then it represents that there are more than one fuzzy length between source node and node j but fuzzy distance along all paths is $\widetilde{e}_j$,so choose any value of i .

Step 4 : Let the destination node (node n) be labelled as $[\widetilde{e}_n, l]$, then the FSL between Source node and destination node is $\widetilde{e}_n$.

Step 5 : Since destination node is labelled as $[\widetilde{e}_n, l]$. So, to find the FSL between source node and destination node, check the label of node l. Let it be $[\widetilde{e}_l, p]$, now check the label of node p and so on. Repeat the same procedure until node 1 is obtained.

Step 6 : The fuzzy shortest path can be obtained by combining all the nodes obtained by the step 5.

4. *Numerical Example*

In order to illustrate the above procedure consider a small network shown in figure, where each arc length is represented as a triangular intuitionistic fuzzy number.

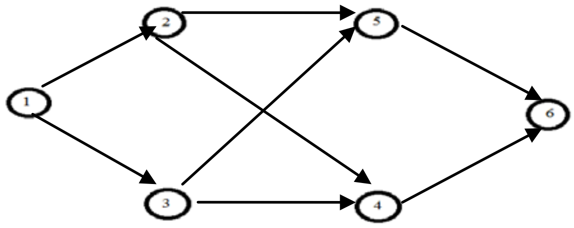

Figure -1

Solution: A network with the triangular Intuitionistic fuzzy arc lengths is shown in figure-1.

Assume the arc lengths as

$$e_{12} = \left(\langle 15,26,37\rangle, \langle 30,39,42\rangle, \langle 43,45,47\rangle\right)$$
$$e_{13} = \left(\langle 22,31,43\rangle, \langle 35,44,46\rangle, \langle 45,48,50\rangle\right)$$

$$e_{24} = \left(\langle 31,37,48\rangle, \langle 41,50,52\rangle, \langle 51,54,56\rangle\right)$$
$$e_{25} = \left(\langle 30,45,51\rangle, \langle 48,52,56\rangle, \langle 53,57,59\rangle\right)$$

$$e_{34} = \left(\langle 12,34,40\rangle, \langle 42,45,50\rangle, \langle 47,52,55\rangle\right)$$
$$e_{35} = \left(\langle 13,17,24\rangle, \langle 20,26,29\rangle, \langle 28,32,34\rangle\right)$$

$$e_{46} = \left(\langle 20,25,31\rangle, \langle 27,33,42\rangle, \langle 35,43,45\rangle\right)$$
$$e_{56} = \left(\langle 22,30,42\rangle, \langle 35,46,49\rangle, \langle 48,51,53\rangle\right)$$

The possible paths and the corresponding path lengths are as follows:

Path P_1:1-2-5-6

$$L_1 = \left(\langle 67,101,130\rangle, \langle 113,137,147\rangle, \langle 144,153,159\rangle\right)$$

Path P_2:1-2-4-6

$$L_2 = \left(\langle 66,88,116\rangle, \langle 98,122,136\rangle, \langle 129,142,148\rangle\right)$$

Path P_3 :1-3-4-6

$$L_3 = \left(\langle 54,90,114\rangle, \langle 104,122,138\rangle, \langle 127,143,150\rangle\right)$$

Path P_4 :1-3-5-6

$$L_4 = \left(\langle 57,78,109\rangle, \langle 90,116,124\rangle, \langle 121,131,137\rangle\right)$$

Since node 6 is the destination node , so n = 6.

Assume $e_1 = \left(\langle 0,0,0\rangle, \langle 0,0,0\rangle, \langle 0,0,0\rangle\right)$

And label the source node (say node 1) as

[$(\langle 0,0,0\rangle,\langle 0,0,0\rangle,\langle 0,0,0\rangle)$, -], the values of e_j ; j = 2,3,4,5,6 can be obtained as follows:

Step 1 : Since only node 1 is the predecessor node of node 2, so putting i = 1 and j = 2 in step2 of the proposed algorithm,the value of e_2 is

$$e_2 = \min\{e_1 \oplus e_{12}\}$$
$$= \min\{(<0,0,0>,<0,0,0>,<0,0,0>) \oplus (<15,26,37>,<30,39,42>,<43,45,47>)\}$$
$$e_2 = (<15,26,37>,<30,39,42>,<43,45,47>)$$

Since minimum occurs corresponding to i = 1 , so label node 2 as

$$[(\langle 15,26,37\rangle,\langle 30,39,42\rangle,\langle 43,45,47\rangle),1].$$

Step 2 : The predecessor node of the node 3 is node 1 so putting i = 1 and j = 3 in step 2 of the algorithm ,the value of e_3 is

$$e_3 = \min\{e_1 \oplus e_{13}\}$$
$$= \min\{(<0,0,0>,<0,0,0>,<0,0,0>) \oplus (<22,31,43>,<35,44,46>,<45,48,50>)\}$$

i.e., $e_3 = (<22,31,43>,<35,44,46>,<45,48,50>)$

Since minimum occurs corresponding to i = 1, so label node 3 as

$$[(\langle 22,31,43\rangle,\langle 35,44,46\rangle,\langle 45,48,50\rangle),1]$$

Step 3 : The predecessor nodes of the node 4 are node 2 and 3, so putting i = 2,3 and j = 4 in Step 2 of the algorithm, the value of e_4 is

$$e_4 = \min\{e_2 \oplus e_{24}, e_3 \oplus e_{34}\}$$
$$= \min\{[(<15,26,37>,<30,39,42>,<43,45,47>) \oplus (<31,37,48>,<41,50,52>,<51,54,56>)], [(<22,31,43>,<35,44,46>,<45,48,50>) \oplus (<12,34,40>,<42,45,50>,<47,52,55>)]\}$$

$$= \min\begin{bmatrix}(\langle 46,63,85\rangle,\langle 71,89,94\rangle,\langle 94,99,103\rangle),\\(\langle 34,65,83\rangle,\langle 77,89,96\rangle,\langle 92,100,105\rangle)\end{bmatrix}$$

$\Re_1 = \frac{1}{3}\{211+251+282\} = 248$ and

$\Re_2 = \frac{1}{3}\{203+254+284\} = 247$

Since $\Re_2 < \Re_1$, *so*

$$e_4 = (\langle 34,65,83\rangle,\langle 77,89,96\rangle,\langle 92,100,105\rangle)$$

Since minimum occurs corresponding to i = 3, so label node 4 as

$$[(\langle 34,65,83\rangle,\langle 77,89,96\rangle,\langle 92,100,105\rangle),3]$$

Step 4 :

The predecessor nodes of the node 5 are node 2 and 3, so putting i = 2,3 and j = 5 in Step 2 of the algorithm , the value of e_5 is

$$e_5 = \min\{e_2 \oplus e_{25}, e_3 \oplus e_{35}\}$$
$$= \min\{[(<15,26,37>,<30,39,42>,<43,45,47>) \oplus (<30,45,51>,<48,52,56>,<53,57,59>)], [(<22,31,43>,<35,44,46>,<45,48,50>) \oplus (<13,17,24>,<20,26,29>,<28,32,34>)]\}$$

$$= \min\begin{bmatrix}(\langle 45,71,88\rangle,\langle 78,91,98\rangle,\langle 96,102,106\rangle),\\(\langle 35,48,67\rangle,\langle 55,70,75\rangle,\langle 73,80,84\rangle)\end{bmatrix}$$

$$\Re_1 = \frac{1}{3}[219+264+292] = 258.33,$$

$$\Re_2 = \frac{1}{3}[163+198+226] = 195.67$$

Since $\Re_2 < \Re_1$, *so*

$$e_5 = (\langle 35,48,67\rangle,\langle 55,70,75\rangle,\langle 73,80,84\rangle)$$

Since minimum occurs corresponding to i = 3, so label node 5 as

$$[(\langle 35,48,67\rangle,\langle 55,70,75\rangle,\langle 73,80,84\rangle),3]$$

Step 5 : The predecessor nodes of the node 6 are node 4 and 5, so putting i = 4,5 and j = 6 in Step 2 of the algorithm , the value of e_6 is

$$e_6 = \min\{e_4 \oplus e_{46}, e_5 \oplus e_{56}\}$$
$$= \min\{[(<34,65,83>,<77,89,96>,<92,100,105>) \oplus (<20,25,31>,<27,33,42>,<35,43,45>)], [(<35,48,67>,<55,70,75>,<73,80,84>) \oplus (<22,30,42>,<35,46,49>,<48,51,53>)]\}$$

$$=\min\begin{bmatrix}(\langle 54,90,114\rangle,\langle 104,122,138\rangle,\langle 127,143,150\rangle),\\(\langle 57,78,109\rangle,\langle 90,116,124\rangle,\langle 121,131,137\rangle)\end{bmatrix}$$

$$\Re_1 = \frac{1}{3}[285+355+402] = 347.33,$$

$$\Re_2 = \frac{1}{3}[268+325+370] = 321$$

Since $\Re_2 < \Re_1$, so

$$e_6 = (\langle 57,78,109\rangle,\langle 90,116,124\rangle,\langle 121,131,137\rangle)$$

Since minimum occurs corresponding to i = 5, so label node 6 as

$$[(\langle 57,78,109\rangle,\langle 90,116,124\rangle,\langle 121,131,137\rangle),5]$$

Since node 6 is the destination node of the given network, so the FSD between node 1 and 6 is $(\langle 57,78,109\rangle,\langle 90,116,124\rangle,\langle 121,131,137\rangle)$.

Now the FSL between node 1 and node 6 can be obtained by using the following procedure. Since node 6 is labelled by

$$[(\langle 57,78,109\rangle,\langle 90,116,124\rangle,\langle 121,131,137\rangle),5]$$

which represents that we are coming from node 5.

Node 5 is labelled by

$\left[\left(\langle 35,48,67\rangle,\langle 55,70,75\rangle,\langle 73,80,84\rangle\right),3\right]$, which represents that we are coming from node 3. Node 3 is labelled by

$\left[\left(\langle 22,31,43\rangle,\langle 35,44,46\rangle,\langle 45,48,50\rangle\right),1\right]$,

which represents that we are coming from node 1. Now the FSL between node 1 and node 6 is obtained by joining all the obtained nodes . Hence the fuzzy shortest path is 1-3-5-6.The FSD and the fuzzy shortest path of all the nodes from node 1 is shown in the following table :

Node No. (j)	$\tilde{e}_j$	FSL between j^{th} and 1^{st} node
2	$(<15,26,37>,<30,39,42>,<43,45,47>)$	$1 \to 2$
3	$(<22,31,43>,<35,44,46>,<45,48,50>)$	$1 \to 3$
4	$(\langle 34,65,83\rangle,\langle 77,89,96\rangle,\langle 92,100,105\rangle)$	$1 \to 3 \to 4$
5	$(\langle 35,48,67\rangle,\langle 55,70,75\rangle,\langle 73,80,84\rangle)$	$1 \to 3 \to 5$
6	$(\langle 57,78,109\rangle,\langle 90,116,124\rangle,\langle 121,131,137\rangle)$	$1 \to 3 \to 5$

Conclusion

Fuzzy shortest path length and shortest path are the useful information for the decision makers. In this paper, we have developed an algorithm for solving Shortest Path Problem on a network with fuzzy arc lengths in intuitionistic sense. An illustrative example is included to demonstrate the proposed method.

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